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Does Quantum Nonlocality Exist? Bell's Theorem and the Many-Worlds Interpretation

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Abstract

Quantum nonlocality may be an artifact of the assumption that observers obey the laws of classical mechanics, while observed systems obey quantum mechanics. I show that, at least in the case of Bell's Theorem, locality is restored if observed and observer are both assumed to obey quantum mechanics, as in the Many-Worlds Interpretation. Using the MWI, I shall show that the apparently "non-local" expectation value for the product of the spins of two widely separated particles — the "quantum" part of Bell's Theorem — is really due to a series of three purely local measurements. Thus, experiments confirming "nonlocality" are actually confirming the MWI.

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Nonlocality is the standard example of a quantum mechanical property not present in classical mechanics. Many papers are published each year (in 1997, four in PRL alone [1]; in 1998, six in PRL alone [2]; in 1999, eight in PRL alone [3]; and three in *Nature* [4]) on the subject of"nonlocality," many (e.g., the papers just cited) showing truly awesome ingenuity. The phenomenon of nonlocality was first discussed in the EPR Experiment [5]. We have two spin 1/2 particles, and the two-particle system is in the rotationally invariant singlet state with zero total spin angular momentum. Thus, if we decide to measure the particle spins in the up-down direction, we would write the wave function of such a state as

$$|\Psi>= \frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}} \tag{1}$$

where the direction of the arrow denotes the direction of spin, and the subscript denotes the particle. If we decide to measure the particle spins in the left-right direction, the wave function would be written in a left-right basis as

$$|\Psi>= \frac{|\leftarrow>_1|\rightarrow>_2 - |\rightarrow>_1|\leftarrow>_2}{\sqrt{2}} \tag{2}$$

Nonlocality arises if and only if we assume that the measurement of the spin of a particle "collapses the wave function" from the linear superposition to *either* $|\uparrow>_1|\downarrow>_2$ *or* $|\downarrow>_1|\uparrow>_2$ in (1). *If* such a collapse occurs, then measuring the spin of particle one would fix the spin of particle two. The spin of particle two would be fixed instantaneously, even if the particles had been allowed to separate to large distances. If at the location of particle one, we make a last minute decision to measure the spin of particle one in the left-right direction rather than the up-down direction, then instantaneously the spin of particle two would be fixed in the opposite direction as particle one — *if* we assume that (2) collapses at the instant we measure the spin of particle one. The mystery of quantum nonlocality lies in trying to understand how particle two changes — instantaneously — in response to what has happened in the location of particle one.

There is no mystery. There is no quantum nonlocality. Particle two *doesn't* know what has happened to particle one when its spin is measured. State transitions are nice and local in quantum mechanics. These statements are true because quantum mechanics tells us that the wave function does *not* collapse when the state of a system is measured. In particular, nonlocality disappears when the Many-Worlds Interpretation (MWI) [6,7,8,16] is adopted. The MWI dispels the mysteries of quantum mechanics. D.N. Page has previously shown [9] how the EPR reality criterion is completely fulfilled by the MWI. I shall extend his analysis, and show how the "quantum" part of Bell's Theorem [10], namely the expectation valve for the product of the spins of the two widely separated electrons, a quantity generally believed to be essentially non-local, actually arises from a series of *local* measurements.

To see how nonlocality disappears, let us analyze the measure of the spins of the two particles from the Many-Worlds perspective. Let $M_i(...)$ denote the initial state of the device which measures the spin of the ith particle. The ellipsis will denote a measurement not yet having been performed. We can for simplicity assume that the apparatus is 100% efficient and that the measurement doesn't change the spin being measured (putting in a more realistic efficiency and taking into account the fact that measurement may affect the spin slightly would complicate the notation but the conclusions would be unchanged). That is, if each particle happens to be in an eigenstate of spin, a measurement of the ith particle changes the measuring device — but not the spin of the particle — as follows:

$$\mathcal{U}_1 M_1(...)|\uparrow>_1 = M_1(\uparrow)|\uparrow>_1, \qquad \mathcal{U}_1 M_1(...)|\downarrow>_1 = M_1(\downarrow)|\downarrow>_1 \tag{3}$$

$$\mathcal{U}_2 M_2(...)|\uparrow>_2 = M_2(\uparrow)|\uparrow>_2, \qquad \mathcal{U}_2 M_2(...)|\downarrow>_2 = M_2(\downarrow)|\downarrow>_2 \tag{4}$$

where $\mathcal{U}_i$ are *linear* operators which generates the change of state in the measurement apparatus, corresponding to the measurement.

In particular, if particle 1 is in an eigenstate of spin up, and particle 2 is in an eigenstate of spin down, then the effect of the $\mathcal{U}_i$'s together is

$$\mathcal{U}_2\mathcal{U}_1 M_1(...)M_2(...)|\uparrow>_1|\downarrow>_2 = M_1(\uparrow)M_2(\downarrow)|\uparrow>_1|\downarrow>_2 \tag{5}$$

even if particles 1 and 2 are light years apart when their spin orientations are measured. Similarly, the result of measuring the ith particle in the eigenstate of spin left would be $\mathcal{U}_i M_i(...)|\leftarrow>_i = M_i(\leftarrow)|\leftarrow>_i$, and for an eigenstate of spin right $\mathcal{U}_i M_i(...)|\rightarrow>_i = M_i(\rightarrow)|\rightarrow>_i$, which will generate equations for spins left and right analogous to eqs. (3) – (5).

Now consider the effect of a measurement on the two particle system in the Bohm state, that is, with total spin zero. This state is (1) or (2) with respect to an up/down or left/right basis respectively. The result is *completely* determined by linearity and the assumed correct measurements on single electrons in eigenstates. For example, the effect of measurements in which both observers happen to choose to measure with respect to the up/down basis is

$$\mathcal{U}_2\mathcal{U}_1 M_2(...)M_1(...)\left[\frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$

$$\mathcal{U}_2 M_2(...)\left[\frac{M_1(\uparrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_1(\downarrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right]$$

$$= \frac{M_2(\downarrow)M_1(\uparrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_2(\uparrow)M_1(\downarrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}} \tag{6}$$

It may appear from eqn. (6) that it is the first measurement to be carried out that determines the split into the two worlds represented by two terms in (6). This is false. In fact, if the measurements are carried out at spacetime events which are spacelike separated, then there is no Lorentz invariant way of determining which measurement was carried out first. At spacelike separation, the measuring operators $\mathcal{U}_1$ and $\mathcal{U}_2$ commute, and so we can equally well perform the measurement of the spins of the electrons in reverse order and obtain the same splits:

$$\mathcal{U}_1\mathcal{U}_2 M_1(...)M_2(...)\left[\frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$

$$\mathcal{U}_1 M_1(...)\left[\frac{M_2(\downarrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_2(\uparrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right]$$

$$= \frac{M_1(\uparrow)M_2(\downarrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - \frac{M_1(\downarrow)M_2(\uparrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}} \tag{7}$$

the last line of which is the same as that of (6), (except for the order of states, which is irrelevant).

The effect of measurements in which both observers happen to choose to measure with respect to the left/right basis is

$$\mathcal{U}_2\mathcal{U}_1 M_2(...)M_1(...)\left[\frac{|\leftarrow>_1|\rightarrow>_2 - |\rightarrow>_1|\leftarrow>_2}{\sqrt{2}}\right] =$$

$$\mathcal{U}_2 M_2(...)\left[\frac{M_1(\leftarrow)|\leftarrow>_1|\rightarrow>_2}{\sqrt{2}} - \frac{M_1(\rightarrow)|\rightarrow>_1|\leftarrow>_2}{\sqrt{2}}\right]$$

$$= \frac{M_2(\rightarrow)M_1(\leftarrow)|\leftarrow>_1|\rightarrow>_2}{\sqrt{2}} - \frac{M_2(\leftarrow)M_1(\rightarrow)|\rightarrow>_1|\leftarrow>_2}{\sqrt{2}} \tag{8}$$

A comparison of (6)/(7) with (8) shows that *if* two spacelike-separated observers fortuitously happen to measure the spins of the two particles in the same direction — whatever this same direction happens to be — both observers will split into two distinct worlds, and in each world the observers will measure opposite spin projections for the electrons. But at each event of observation, *both* of the two possible outcomes of the measurement will be obtained. Locality is preserved, because indeed both outcomes are obtained in total independence of the outcomes of the other measurement. The linearity of the operators $\mathcal{U}_1$ and $\mathcal{U}_2$ forces the perfect anti-correlation of the spins of the particles in each world. Since the singlet state is rotationally invariant, the same result would be obtained whatever direction the observers happened to choose to measure the spins.

In the EPR experiment, there is actually a third measurement: the comparison of the two observations made by the spatially separated observers. In fact, the relative directions of the two spin measurements have no meaning without this third measurement. Once again, it is easily seen that initialization of this third measurement by the two previous measurements, plus linearity implies that this third measurement will confirm the split into two worlds. In the usual analysis, this third measurement is not considered a quantum measurement at all, because the first measurements are considered as transferring the data from the quantum to the classical regime. But in the MWI, there is no classical regime; the comparison of the data in two macroscopic devices is just a much a quantum interaction as the original setting up of the singlet state. Furthermore, this ignored third measurement is actually of crucial importance: it is performed *after* information about the orientation of the second device has been carried back to the first device (at a speed less than light!). The orientation is coded with correlations of the spins of both electrons, and these correlations (and the linearity of all operators) will force the third measurement to respect the original split. These correlations have not been lost, for no measurement reduces the wave function: the minus sign between the two worlds is present in all eqns. (1) — (8).

To see explicitly how this third measurement works, represent the state of the comparison apparatus by $M_c[(...)_1(...)_2]$, where the first entry measures the record of the apparatus measuring the first particle, and the second entry measures the record of the apparatus measuring the second particle. Thus, the third measurement acting on eigenstates of the spin-measurement devices transforms the comparison apparatus as follows:

$$\mathcal{U}_c M_c[(...)_1(...)_2]M_1(\uparrow) = M_c[(\uparrow)_1(...)_2]M_1(\uparrow)$$

$$\mathcal{U}_c M_c[(...)_1(...)_2]M_1(\downarrow) = M_c[(\downarrow)_1(...)_2]M_1(\downarrow)$$

$$\mathcal{U}_c M_c[(...)_1(...)_2]M_2(\uparrow) = M_c[(...)_1(\uparrow)_2]M_2(\uparrow)$$

$$\mathcal{U}_c M_c[(...)_1(...)_2]M_2(\downarrow) = M_c[(...)_1(\downarrow)_2]M_2(\downarrow)$$

where for simplicity I have assumed the spins will be measured in the up or down direction. Then for the state (1), the totality of the three measurements together — the two measurements of the particle spins followed by the comparison measurement — is

$$\mathcal{U}_c\mathcal{U}_2\mathcal{U}_1 M_c[(...)_1(...)_2]M_2(...)M_1(...)\left[\frac{|\uparrow>_1|\downarrow>_2 - |\downarrow>_1|\uparrow>_2}{\sqrt{2}}\right] =$$

$$= M_c[(\uparrow)_1(\downarrow)_2]\frac{M_2(\downarrow)M_1(\uparrow)|\uparrow>_1|\downarrow>_2}{\sqrt{2}} - M_c[(\downarrow)_1(\uparrow)_2]\frac{M_2(\uparrow)M_1(\downarrow)|\downarrow>_1|\uparrow>_2}{\sqrt{2}}$$

Heretofore I have assumed that the two observers have chosen to measure the spins in the same direction. For observers who make the decision of which direction to measure the spin in the instant before the measurement, most of the time the two directions will not be the same. The experiment could be carried out by throwing away all observations except those in which the chosen directions happened to agree within a predetermined tolerance. But this would waste most of the data. The Aspect-Clauser-Freedman Experiment [11] is designed to use more of the data by testing Bell's Inequality for the expectation value of the product of the spins of the two electrons with the spin of one electron being measured in direction $\hat{\mathbf{n}}_1$, and the spin of the other in direction $\hat{\mathbf{n}}_2$. If the spins are measured in units of $\hbar/2$, the standard QM expectation value for the product is

$$< \Psi|(\hat{\mathbf{n}}_1 \cdot \sigma_1)(\hat{\mathbf{n}}_2 \cdot \sigma_2)|\Psi >= -\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2 \tag{9}$$

where $|\Psi>$ is the singlet state (1)/(2). In particular, $\hat{\mathbf{n}}_1 = \hat{\mathbf{n}}_2$ is the assumed set-up of the previous discussion. Since the MWI shows that local measurements in this case always gives +1 for one electron and -1 for the other, the product of the two is always -1 in all worlds, and thus the expectation value for the product is -1, in complete agreement with (9).

To show how (9) comes about by local measurements splitting the universe into distinct worlds, I follow [12] and write the singlet state (1)/(2) with respect to some basis in the $\hat{\mathbf{n}}_1$ direction as

$$|\Psi>= (1/\sqrt{2})(|\hat{\mathbf{n}}_1,\uparrow>_1 |\hat{\mathbf{n}}_1,\downarrow>_2 - |\hat{\mathbf{n}}_1,\downarrow>_1 |\hat{\mathbf{n}}_1,\uparrow>_2) \tag{10}$$

Let another direction $\hat{\mathbf{n}}_2$ be the polar axis, with θ the polar angle of $\hat{\mathbf{n}}_1$ relative to $\hat{\mathbf{n}}_2$. Without loss of generality, we can choose the other coordinates so that the azimuthal angle of $\hat{\mathbf{n}}_1$ is zero. Standard rotation operators for spinor states then give [12]

$$|\hat{\mathbf{n}}_1,\uparrow>_2 = (\cos\theta/2)|\hat{\mathbf{n}}_2,\uparrow>_2 + (\sin\theta/2)|\hat{\mathbf{n}}_2,\downarrow>_2$$

$$|\hat{\mathbf{n}}_1,\downarrow>_2 = -(\sin\theta/2)|\hat{\mathbf{n}}_2,\uparrow>_2 + (\cos\theta/2)|\hat{\mathbf{n}}_2,\downarrow>_2$$

which yields

$$\begin{aligned}|\Psi>= (1/\sqrt{2})[&-(\sin\theta/2)|\hat{\mathbf{n}}_1,\uparrow>_1 |\hat{\mathbf{n}}_2,\uparrow>_2 + (\cos\theta/2)|\hat{\mathbf{n}}_1,\uparrow>_1 |\hat{\mathbf{n}}_2,\downarrow>_2 \\ &-(\cos\theta/2)|\hat{\mathbf{n}}_1,\downarrow>_1 |\hat{\mathbf{n}}_2,\uparrow>_2 - (\sin\theta/2)|\hat{\mathbf{n}}_1,\downarrow>_1 |\hat{\mathbf{n}}_2,\downarrow>_2]\end{aligned} \tag{11}$$

In other words, if the two devices measure the spins in arbitrary directions, there will be a split into *four* worlds, one for each possible permutation of the electron spins. Just as in the case with $\hat{\mathbf{n}}_1 = \hat{\mathbf{n}}_2$, normalization of the devices on eigenstates plus linearity forces the devices to split into all of these four worlds, which are the only possible worlds, since each observer must measure the electron to have spin $+1$ or -1.

The squares of the coefficients in (11) are proportional to the number of worlds wherein each respective possibility occurs, these possibilities being determined by the chosen experimental arrangement. This is most easily seen using Deutsch's MWI derivation [13,16] of the Born Interpretation (BI). DeWitt and Graham [7] originally deduced the BI using the relative frequency theory of probability [14,15], and this derivation is open to the standard objections to the frequency theory [14,15]. Deutsch instead derives the BI using the Principle of Indifference of the classical/a priori theory of probability [14]. According to the Principle of Indifference, the probability of an event is the number of times the event occurs in a collection of equipossible cases divided by the total number of equipossible cases. Thus, the probability is 1/6 that a single die throw will result in a 5, because there are 6 equipossible sides that could appear, of which the 5 is exactly 1.

Deutsch assumes the Principle of Indifference applies to any experimental arrangement in which the expansion of the wave function $|\Psi>$ in terms of the orthonormal basis vectors of the experiment (the interpretation basis) give equal coefficients for each term in the expansion. For example, both (1) and (2) are two such expansions, because in both cases the coefficients of each of the two terms is $1/\sqrt{2}$. The Principle of Indifference thus says that each of the two possibilities is equally likely in either the experimental arrangement (1), or in the interpretation basis (2). Equivalently, there are an equal number of worlds corresponding to each term in either (1) or (2), since in the MWI "equally possible" means "equal number of worlds" (equal relative to a preset experimental arrangement).

Deutsch shows [13,16] that if the squares of the coefficients in the interpretation basis of an experiment are rational, then a new experimental arrangement can be found in which the coefficients are equal in the new interpretation basis. Applying the Principle of Indifference to this new set of coefficients yields the BI for the coefficients in the original basis. Continuity in the Hilbert space of wave functions yields the BI for irrational coefficients (although it is a presupposition of the MWI that only coefficients with rational squares are allowed since irrational squares would imply an irrational number of worlds). In particular, the percentage of worlds with the value of a given basis vector is given by the square of the coefficient.

The expectation value (9) for the product of the spins is just the sum of each outcome, multiplied respectively by probabilities of each of the four possible outcomes:

$$(+1)(+1)P_{\uparrow\uparrow} \ + \ (+1)(-1)P_{\uparrow\downarrow} \ + \ (-1)(+1)P_{\downarrow\uparrow} \ + \ (-1)(-1)P_{\downarrow\downarrow} \tag{12}$$

where $P_{\uparrow\downarrow}$ is the relative number of worlds in which the first electron is measured spin up, and the second electron spin down, and similarly for the other P's. Inserting these relative numbers — the squares of the coefficients in (11) — into (12) gives the expectation value:

$$= \frac{1}{2}\sin^2\theta/2 - \frac{1}{2}\cos^2\theta/2 - \frac{1}{2}\cos^2\theta/2 + \frac{1}{2}\sin^2\theta/2 = -\cos\theta = -\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2 \tag{13}$$

which is the quantum expectation value (9).

Once again it is essential to keep in mind the third measurement that compares the results of the two measurements of the spins, and by bringing the correlations between the worlds back to the same location, defines the relative orientation of the previous two measurements, and in fact determines whether there is a twofold or a fourfold split. The way the measurement of (9) is actually carried out in the Aspect-Clauser-Freedman Experiment is to let θ be random in any single run, and for the results of each fixed θ from a series of runs be placed in separate bins. This separation requires the third measurement, and this local comparison measurement retains the correlations between the spins. The effect of throwing away this correlation information would be equivalent to averaging over all θ in the computation of the expectation value: the result is $\int_0^\pi < \Psi|(\hat{\mathbf{n}}_1 \cdot \sigma_1)(\hat{\mathbf{n}}_2 \cdot \sigma_2)|\Psi > \, d\theta = 0$; i.e., the measured spin orientations of the two electrons are completely uncorrelated. This is what we would expect if each measurement of the electron spins is completely local, which in fact they are. There is no quantum nonlocality.

Bell's results [10,15] lead one to think otherwise. But Bell made the tacit assumption that each electron's wave function is reduced by the measurement of its spin. Specifically, he assumed that the first electron's spin was determined by the measurement direction $\hat{\mathbf{n}}_1$ and the value some local hidden variable parameters λ_1: the first electron's spin is given by a function $A(\hat{\mathbf{n}}_1, \lambda_1)$. The second electron's spin is given by an analogous function $B(\hat{\mathbf{n}}_2, \lambda_2)$, and so the hidden variable expectation value for the product of the spins would not be (13) but instead

$$\int \rho(\lambda_1, \lambda_2) A(\hat{\mathbf{n}}_1, \lambda_1) B(\hat{\mathbf{n}}_2, \lambda_2) \, d\lambda_1 \, d\lambda_2 \tag{14}$$

where $\rho(\lambda_1, \lambda_2)$ is the joint probability distribution for the hidden variables. By comparing a triple set of directions $(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3)$, Bell derived the inequality $|P(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) - P(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_3)| \leq 1 + P(\hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3)$, which for certain choices of the triple, is inconsistent with the quantum mechanical (9); i.e., $\hat{\mathbf{n}}_1 = \hat{\mathbf{n}}_2 - \hat{\mathbf{n}}_3/|\hat{\mathbf{n}}_2 - \hat{\mathbf{n}}_3|$ yields $\sqrt{2} \leq 1$ if we assume the MW result (9)/(13), which is the quantum part of Bell's Theorem.

But (14) assumes that the spin of each particle is a *function* of $\hat{\mathbf{n}}_i$ and λ_i; that is, it assumes the spin at a location is *single-valued*. This is explicitly denied by the MWI, as one can see by letting λ_i be the spatial coordinates of the ith electron. Bell's analysis tacitly assumes that the macroscopic world is a single-valued world like classical mechanics.

The automatic elimination of action at a distance by the MWI is a powerful argument for the validity of the MWI, for assuming that both single electrons and many-atom measuring devices are described by multivalued quantum states.

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